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Jordanian quantum deformations of $D = 4$ anti-de Sitter and Poincaré superalgebras

A. Borowiec[1], J. Lukierski[1,a], V.N. Tolstoy[1,2]

[1] Institute for Theoretical Physics, University of Wrocław, pl. Maxa Borna 9, 50-205 Wrocław, Poland
[2] Institute of Nuclear Physics, Moscow State University, 119 992 Moscow, Russia

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Abstract. We consider a superextension of the extended Jordanian twist, describing the non-standard quantization of the anti-de Sitter (AdS) superalgebra $\mathfrak{osp}(1|4)$ in the form of a Hopf superalgebra. The super-Jordanian twisting function and corresponding basic coproduct formulae for the generators of $\mathfrak{osp}(1|4)$ are given in explicit form. A non-linear transformation of the classical superalgebra basis not modifying the defining algebraic relations but simplifying coproducts and antipodes is proposed. Our physical application is in the interpretation of the new super-Jordanian deformation of the $\mathfrak{osp}(1|4)$ superalgebra as deformed $D = 4$ AdS supersymmetries. Subsequently we perform a suitable contraction of the quantum Jordanian AdS superalgebra and obtain a new κ-deformation of the $D = 4$ Poincaré superalgebra, with the bosonic sector describing the light-cone κ-deformation of the Poincaré symmetries.

1 Introduction

The main aims of studying the quantum deformations of space-time Lie algebras and Lie superalgebras is to provide the geometric origin of deformed relativistic symmetries, non-commutative space-times and their corresponding supersymmetric extensions. The quantum deformations of Poincaré, AdS and conformal space-time symmetries in $D = 4$ were already extensively studied (see e.g. [1–13]). In particular for the Poincaré and conformal algebras one can introduce the deformations parametrized by the geometric mass parameter κ, i.e. described as so-called κ-deformations (see e.g. [1, 5, 6, 8–13]) which were also used for the introduction of κ-deformed field theories (see e.g. [14–18]). Such deformations introduce in a geometric way the third fundamental parameter κ in physics (besides $\hbar$ and c) which can be linked with the Planck mass and quantum gravity [19, 20]. It appears that in the general case, e.g. for the $D = 4$ conformal algebra, one can introduce several mass-like deformation parameters [12].

It is well known that in the recent quarter of the last century the new unified models of fundamental interactions are supersymmetric (e.g. supergravities, superstrings, super-p-branes, super-D-branes, M-theory). In particular, we stress that the non-commutative space-times describing D-brane world-volume coordinates in a Kalb–Ramond two-tensor background [21] in fact should be extended supersymmetrically (see e.g. [22–25]). We argue therefore that if the notion of non-commutative geometry and quantum groups are applicable to the present supersymmetric framework of fundamental interactions, the non-commutative superspaces as well as the quantum supersymmetries should be studied. In this paper we limit ourselves to the case of "physical" $D = 4$ SUSY case; the application to e.g. M-theory requires the consideration of deformed supersymmetries and superspaces for all $D \leq 11$.

For $D = 4$ supersymmetries only the standard κ-deformation of the Poincaré superalgebra has been studied [26–29] which was obtained as the quantum AdS contraction of the Drinfeld–Jimbo q-deformation of $\mathfrak{osp}(1|4)$, in the contraction limit $\lim_{\substack{q\to 1\\ R\to\infty}} R \ln q = \kappa^{-1}$, where R is the AdS radius. In such a limit the classical r-matrix

$$r = \frac{1}{\kappa} \sum_{i=1}^{3} N_i \wedge P_i, \tag{1.1}$$

describing the standard κ-deformed $D = 4$ Poincaré algebra, was supersymmetrized. We recall that the elements $M_j = \frac{1}{2}\epsilon_{jkl} M_{kl}$, $N_j = M_{0j}$, P_k, P_0 $(j = 1, 2, 3)$ generating the Poincaré algebra $\mathcal{P}(3,1) = \{M_{\mu\nu}, P_\mu \,|\, \mu, \nu = 0, \ldots, 4\}$ satisfy the standard commutation relations:

$$\begin{aligned}
&[M_j,\, M_k] = \mathrm{i}\epsilon_{jkl}\, M_l, && [M_j,\, N_k] = \mathrm{i}\epsilon_{jkl}\, N_l,\\
&[N_j,\, N_k] = -\mathrm{i}\epsilon_{jkl}\, M_l, && [M_j,\, P_k] = \mathrm{i}\epsilon_{jkl}\, P_l,\\
&[M_j,\, P_0] = 0, && [N_j,\, P_k] = -\mathrm{i}\delta_{jk}\, P_0,\\
&[N_j,\, P_0] = -\mathrm{i}P_j, && [P_\mu,\, P_\nu] = 0.
\end{aligned} \tag{1.2}$$

In this paper we consider another non-standard deformation of the $\mathfrak{osp}(1|4)$ superalgebra with the classical

[a] e-mail: lukier@ift.uni.wroc.pl

r-matrix of Jordanian type, satisfying the graded classical Yang–Baxter equation [30, 31]. We show that in the quantum contraction limit $R \to \infty$ the fundamental mass parameter κ entering into our new κ-deformed $D = 4$ super-Poincaré algebra is defined as follows:

$$\lim_{\substack{\xi \to 0 \\ R \to \infty}} \xi R = \frac{\mathrm{i}}{\kappa}, \tag{1.3}$$

where ξ denotes the suitably chosen dimensionless parameter describing the non-standard deformation of $\mathfrak{osp}(1|4)$. In such a way we obtain an alternative κ-deformation of the $D = 4$ super-Poincaré algebra described infinitesimally by the supersymmetrization of the following r-matrix for the light-cone κ-deformation of the $D = 4$ Poincaré algebra [8, 12]:

$$r = \frac{1}{\kappa} \left(P_1 \wedge (N_1 + M_2) + P_2 \wedge (N_2 - M_1) + P_+ \wedge N_3 \right), \tag{1.4}$$

where $P_+ = P_0 + P_3$. The quantum deformation of relativistic symmetries generated by the r-matrix (1.4) describes the κ-deformed Minkowski space with the “quantized” light-cone direction $x_+ = x_0 + x_3$ [9, 10]. We point out that the light-cone κ-Poincaré algebra has been introduced firstly in a particular basis under the name of the null-plane quantum Poincaré algebra [10].

In the general case the classical r-matrices of Jordanian type for any simple Lie algebra depend on several deformation parameters, $\xi_1, \xi_2, \ldots, \xi_n$, and it is a sum of the one-parameter classical r-matrices of Jordanian type [30,32,33], where the one-parameter classical r-matrix has the form

$$r(\xi) = \xi \left(h_{\gamma_0} \wedge e_{\gamma_0} + \sum_{i=1}^{N} e_{\gamma_i} \wedge e_{\gamma_{-i}} \right), \tag{1.5}$$

provided that the generators $(h_{\gamma_0}, e_{\gamma_0}, e_{\gamma_{\pm i}})$ satisfy the relations [30][1]

$$\begin{aligned} &[h_{\gamma_0}, e_{\gamma_0}] = e_{\gamma_0}, \quad [e_{\gamma_i}, e_{\gamma_{-j}}] = \delta_{ij}\, e_{\gamma_0}, \\ &[h_{\gamma_0}, e_{\gamma_i}] = (1 - t_{\gamma_i})\, e_{\gamma_i}, \quad [h_{\gamma_0}, e_{\gamma_{-i}}] = t_{\gamma_i}\, e_{\gamma_{-i}} \\ &\qquad (t_{\gamma_i} \in \mathbb{C}), \\ &[e_{\gamma_{\pm i}}, e_{\gamma_0}] = 0, \quad [e_{\gamma_{\pm i}}, e_{\gamma_{\pm j}}] = 0. \end{aligned} \tag{1.6}$$

In the case of Lie superalgebras the terms constructed from odd (fermionic) generators should be added in (1.5) [30]. For the Lie superalgebra $\mathfrak{osp}(1|4)$ the general classical r-matrix of Jordanian type is of two-parameter type, $r_{\mathrm{AdS}}(\xi_1, \xi_2)$. It turns out that the AdS contraction limit (1.2) gives a non-trivial result provided $\xi = \xi_1 = \xi_2$, and it describes our new κ-deformation of the $D = 4$ super-Poincaré algebra.

Recently in [34] by completing earlier results from [35] (see also [36]) there was obtained a twist quantization of $\mathfrak{osp}(1|2)$, describing the κ-deformation of the $D = 1$ conformal superalgebra, which can be interpreted as the deformation of the $D = 2$ AdS superalgebra. Similarly, in this paper we complete the results presented in [31] and consider the twist quantization of $\mathfrak{osp}(1|4)$ with the physical application to AdS supersymmetry and its super-Poincaré limit.

The plan of our paper is the following. In Sect. 2 we present the mathematical (Cartan–Weyl) basis of $\mathfrak{osp}(1|4)$ and consider two corresponding Jordanian type classical r-matrices. In Sect. 3 we present the two-parameter quantization of these classical r-matrices with the deformation modifying only the coalgebra sector, and calculate the basic coproducts for the $\mathfrak{osp}(1|4)$ generators. Further, employing the formulas recently proposed in [30] we introduce a new more suitable basis for the superalgebra $\mathfrak{osp}(1|4)$. In Sect. 4 we shall introduce the AdS physical basis and perform the AdS contraction, introducing the mass-like deformation parameter κ. In such a limit we obtain the Jordanian type classical r-matrices and the twisting two-tensors for the Poincaré and super-Poincaré algebras. In Sect. 5 we comment on κ-deformations of $N = 1$ Poincaré supersymmetries and on deformed N-extended AdS supersymmetries.

2 Cartan–Weyl basis and Jordanian type classical r-matrices of $\mathfrak{osp}(1|4)$

In order to obtain compact formulas describing the commutation relations for generators of the orthosymplectic superalgebra $\mathfrak{osp}(1|4)$ we use embedding of this superalgebra in the general linear superalgebra $\mathfrak{gl}(1|4)$. For convenience we consider the general case of $\mathfrak{osp}(1|2n)$ [37] embedded in $\mathfrak{gl}(1|2n)$. Let a_{ij} $(i, j = 0, \pm1, \pm2, \ldots, \pm n)$ be a standard basis[2] of the superalgebra $\mathfrak{gl}(1|2n)$ with the standard supercommutation relations

$$\begin{aligned} [a_{ij}, a_{kl}] &:= a_{ij} a_{kl} - (-1)^{\theta_{ij}\theta_{kl}} a_{kl}, \\ a_{ij} &= \delta_{jk} a_{il} - (-1)^{\theta_{ij}\theta_{kl}} \delta_{il} a_{kj}, \end{aligned} \tag{2.1}$$

where $\theta_{ij} = 1$ when one index i or j is equal to 0 and another takes any value $\pm1, \ldots, \pm n$; $\theta_{ij} = 0$ in the remaining cases. The superalgebra $\mathfrak{osp}(1|2n)$ is embedded in $\mathfrak{gl}(1|2n)$ as a linear envelope of the following generators:
(i) the even (boson) generators spanning the symplectic algebra $\mathfrak{sp}(2n)$:

$$e_{ij} := a_{i-j} + \mathrm{sign}(ij)\, a_{j-i} = \mathrm{sign}(ij)\, e_{ji} \quad (i, j = \pm1, \pm2, \ldots, \pm n); \tag{2.2}$$

(ii) the odd (fermion) generators extending $\mathfrak{sp}(2n)$ to $\mathfrak{osp}(1|2n)$:

$$e_{0i} := a_{0-i} + \mathrm{sign}(i)\, a_{i0} = \mathrm{sign}(i)\, e_{i0} \quad (i = \pm1, \pm2, \ldots, \pm n). \tag{2.3}$$

[1] The formulae (1.5) and (1.6) generalize the considerations presented in [32,33] by describing the classical r-matrices with the support $\{h_{\gamma_0}, e_{\gamma_0}, e_{\gamma_{\pm i}}\}$ which does not necessarily belong to the Borel subalgebra.

[2] This basis can be realized by graded $(2n + 1) \times (2n + 1)$-matrices.

We also set $e_{00} = 0$ and introduce the sign function: $\operatorname{sign} x = 1$ if a real number $x \geq 0$ and $\operatorname{sign} x = -1$ if $x < 0$. One can check that the elements (2.2) and (2.3) satisfy the following relations:

$$[e_{ij},\, e_{kl}] = \delta_{j-k} e_{il} + \delta_{j-l} \operatorname{sign}(kl)\, e_{ik} \\ -\delta_{i-l} e_{kj} - \delta_{i-k} \operatorname{sign}(kl)\, e_{lj}, \tag{2.4}$$

$$[e_{ij},\, e_{0k}] = \delta_{j-k} \operatorname{sign}(k)\, e_{i0} - \delta_{i-k} e_{0j}, \tag{2.5}$$

$$\{e_{0i}, e_{0k}\} = \operatorname{sign}(i)\, e_{ik} \tag{2.6}$$

for all $i, j, k, l = \pm 1, \pm 2, \ldots, \pm n$, where the bracket $\{\cdot, \cdot\}$ means the anticommutator.

In our case of $\mathfrak{osp}(1|4)$ we have $n = 2$. The 24 elements e_{ij} $(i, j = 0, \pm 1, \pm 2)$ are not linearly independent (we have 10 constraints, for example, $e_{1-2} = -e_{-21}$) and we can choose from them the Cartan–Weyl basis as follows:

(a) the raising generators :

$$e_{1-2},\ e_{12},\ e_{11},\ e_{22},\ e_{01},\ e_{02}; \tag{2.7}$$

(b) the lowering generators :

$$e_{2-1},\ e_{-2-1},\ e_{-1-1},\ e_{-2-2},\ e_{-10},\ e_{-20}; \tag{2.8}$$

(c) the Cartan generators :

$$h_1 := e_{1-1},\ \ h_2 := e_{2-2}. \tag{2.9}$$

In accordance with [30,31] the general formula for the Jordanian type classical r-matrix of $\mathfrak{osp}(1|4)$ is given as follows:

$$r_{1,2}(\xi_1, \xi_2) = r_1(\xi_1) + r_2(\xi_2), \tag{2.10}$$

where the classical r-matrices of Jordanian type $r_1(\xi_1)$ and $r_2(\xi_2)$ have the form

$$r_1(\xi_1) = \xi_1 \left(\frac{1}{2}\, e_{1-1} \wedge e_{11} + e_{1-2} \wedge e_{12} - 2 e_{01} \otimes e_{01} \right), \tag{2.11}$$

$$r_2(\xi_2) = \xi_2 \left(\frac{1}{2}\, e_{2-2} \wedge e_{22} - 2 e_{02} \otimes e_{02} \right). \tag{2.12}$$

Below we shall describe the twist quantization generated by the classical r-matrix (2.10).

3 Jordanian type deformations of $\mathfrak{osp}(1|4)$

In accordance with the general scheme [30] the complete twisting two-tensor $F(\xi_1, \xi_2)$ corresponding to the resulting Jordanian type r-matrix (2.10) is given as follows:

$$F(\xi_1, \xi_2) = \tilde{F}_{22}(\xi_2) F_{11}(\xi_1), \tag{3.1}$$

where $F_{11}(\xi_1)$ is the twisting two-tensor corresponding to the classical r-matrix (2.11), and $\tilde{F}_{22}(\xi_2)$ is the transformed twisting two-tensor corresponding to the classical r-matrix (2.12) (see below the formulas (3.12)). Thus we can implement full deformation in two steps corresponding to two terms in the formula (2.10).

The first step of Jordanian type deformation. The twisting two-tensor $F_{11}(\xi_1)$ corresponding to the r-matrix (2.11) has the form

$$F_{11}(\xi_1) = \mathfrak{F}_{11}(\xi_1) F_{\sigma_{11}}, \tag{3.2}$$

where the two-tensor $F_{\sigma_{11}}$ is the Jordanian twist [38] and $\mathfrak{F}_{11}$ is the superextension of the Jordanian twist. These two-tensors are given by the formulas

$$F_{\sigma_{11}} = e^{e_{1-1} \otimes \sigma_{11}}, \tag{3.3}$$

$$\mathfrak{F}_{11}(\xi_1) = \exp\left(\xi_1 e_{1-2} \otimes e_{12} e^{-\sigma_{11}}\right) \mathfrak{F}_{01}, \tag{3.4}$$

where the first factor on the RHS of (3.4) describes the extended Jordanian twist [32], and [34]

$$\mathfrak{F}_{01} = \sqrt{\frac{(1 + e^{\sigma_{11}}) \otimes (1 + e^{\sigma_{11}})}{2(1 + e^{\sigma_{11}} \otimes e^{\sigma_{11}})}} \\ \times \left(1 - 2\xi_1 \frac{e_{01}}{1 + e^{\sigma_{11}}} \otimes \frac{e_{01}}{1 + e^{\sigma_{11}}} \right), \tag{3.5}$$

$$\sigma_{11} = \frac{1}{2} \ln(1 + \xi_1 e_{11}). \tag{3.6}$$

We shall not provide here the explicit formulas of the twisted coproduct $\Delta_{\xi_1}(x) := F_{11}(\xi_1) \Delta(x) F_{11}^{-1}(\xi_1)$ and the corresponding antipode $S_{\xi_1}(x)$ for the generators $x = e_{ik}$, since these formulas are intermediate in our scheme[3].

Let us introduce the new generators in $U(\mathfrak{osp}(1|4))$ by the inner automorphism firstly proposed in [30],

$$w_{\xi_1} := \sqrt{u(\mathfrak{F}_{11}(\xi_1))} \\ = \exp\left(\frac{\xi_1\, \sigma_{11}\, e_{1-2}\, e_{12}}{1 - e^{2\sigma_{11}}} \right) \exp\left(\frac{1}{4}\, \sigma_{11} \right), \tag{3.7}$$

where $u(\mathfrak{F}_{11}(\xi_1))$ is the Hopf "folding" of the two-tensor (3.4): $u(\mathfrak{F}_{11}(\xi_1)) = ((S_{\xi_1} \otimes \mathrm{Id}) \mathfrak{F}_{11}(\xi_1)) \circ 1$. We postulate the similarity map $\tilde{e}_{ik} := w_{\xi_1} e_{ik} w_{\xi_1}^{-1}$ preserving the defining relations (2.4)–(2.6).

The twisted coproducts for the elements $\tilde{e}_{ik}$ are given by simple and uniform formulas:

$$\Delta_{\xi_1}(e^{\pm \tilde{\sigma}_{11}}) = e^{\pm \tilde{\sigma}_{11}} \otimes e^{\pm \tilde{\sigma}_{11}}, \tag{3.8}$$

$$\Delta_{\xi_1}(\tilde{e}_{ik}) = \tilde{e}_{ik} \otimes 1 + e^{\tilde{\sigma}_{11}} \otimes \tilde{e}_{ik} \tag{3.9}$$

for $(ik) = (1-2)$, (12), (01) and

$$\Delta_{\xi_1}(\tilde{e}_{1-1}) = \tilde{e}_{1-1} \otimes e^{-2\tilde{\sigma}_{11}} + 1 \otimes \tilde{e}_{1-1} \\ + \xi_1 \left(\tilde{e}_{12} \wedge \tilde{e}_{1-2} + \tilde{e}_{01} \otimes \tilde{e}_{01} \right) \left(e^{-\tilde{\sigma}_{11}} \otimes e^{-2\tilde{\sigma}_{11}} \right). \tag{3.10}$$

The twisted Hopf structure of the subalgebra $\widetilde{\mathfrak{osp}}_2(1|2) := w_{\xi_1} \mathfrak{osp}_2(1|2) w_{\xi_1}^{-1} \subset \widetilde{\mathfrak{osp}}(1|4) := w_{\xi_1} \mathfrak{osp}(1|4) w_{\xi_1}^{-1}$, generated by the elements $\tilde{e}_{22}$, $\tilde{e}_{02}$, $\tilde{e}_{2-2}$, $\tilde{e}_{-2-2}$, $\tilde{e}_{-20}$, is primitive, i.e.

$$\Delta_{\xi_1}(\tilde{e}_{ik}) = \tilde{e}_{ik} \otimes 1 + 1 \otimes \tilde{e}_{ik} \tag{3.11}$$

[3] The coproduct formulas for the generators spanning the classical r-matrix (2.11) can be found in [30].

for $(ik) = (22), (02), (2-2), (-2-2), (-20)$. This is not valid for the initial subalgebra $\mathfrak{osp}_2(1|2)$ what provides a main reason for the introduction of the similarity transformation (3.7) [30]. The formulas for coproducts of the negative generator $\tilde{e}_{2-1}$, $\tilde{e}_{-2-1}$ and $\tilde{e}_{-10}$ have a more complicated form and we do not give them here.

The second step of Jordanian type deformation. Since the subalgebra $\widetilde{\mathfrak{osp}}_2(1|2)$ is not deformed, we can use the results of [34]. Namely, we apply the twisting two-tensor

$$\tilde{F}_{22}(\xi_2) = (w_{\xi_1}\otimes w_{\xi_1})\,\mathfrak{F}_{02}(\xi_2)\,e^{e_{2-2}\otimes\sigma_{22}}\left(w_{\xi_1}^{-1}\otimes w_{\xi_1}^{-1}\right) = \tilde{\mathfrak{F}}_{02}(\xi_2)\,e^{\tilde{e}_{2-2}\otimes\tilde{\sigma}_{22}}, \quad (3.12)$$

where

$$\tilde{\mathfrak{F}}_{02} = \sqrt{\frac{(1+e^{\tilde{\sigma}_{22}})\otimes(1+e^{\tilde{\sigma}_{22}})}{2(1+e^{\tilde{\sigma}_{22}}\otimes e^{\tilde{\sigma}_{22}})}} \times\left(1-2\xi_2\,\frac{\tilde{e}_{02}}{1+e^{\tilde{\sigma}_{22}}}\otimes\frac{\tilde{e}_{02}}{1+e^{\tilde{\sigma}_{22}}}\right), \quad (3.13)$$

$$\tilde{\sigma}_{22} = \frac{1}{2}\ln(1+\xi_2\tilde{e}_{22}) \quad (3.14)$$

to all generators of the ξ_1-deformed superalgebra $\widetilde{\mathfrak{osp}}(1|4)$.

The twisted coproduct

$$\Delta_{\xi_1\xi_2}(\,\cdot\,) := F_{22}(\xi_2)\Delta_{\xi_1}(\,\cdot\,)F_{22}^{-1}(\xi_2)$$

for the elements $\tilde{e}_{ik}$ belonging to the Borel subalgebra of $\mathfrak{osp}(1|4)$ are given by the formulas

$$\Delta_{\xi_1\xi_2}(e^{\pm\tilde{\sigma}_{11}}) = e^{\pm\tilde{\sigma}_{11}}\otimes e^{\pm\tilde{\sigma}_{11}}, \quad (3.15)$$

$$\Delta_{\xi_1\xi_2}(\tilde{e}_{12}) = \tilde{e}_{12}\otimes e^{\tilde{\sigma}_{22}} + e^{\tilde{\sigma}_{11}}\otimes\tilde{e}_{12}, \quad (3.16)$$

$$\begin{aligned}\Delta_{\xi_1\xi_2}(\tilde{e}_{01}) &= \tilde{e}_{01}\otimes 1 + e^{\tilde{\sigma}_{11}}\otimes\tilde{e}_{01}\\ &+\xi_2\left(\tilde{e}_{12}\otimes\tilde{e}_{02} - \tilde{e}_{02}e^{\tilde{\sigma}_{11}}\otimes\tilde{e}_{12}\right)\tilde{\Omega} \quad (3.17)\\ &-\xi_2^2\left(\tilde{e}_{12}\tilde{e}_{02}\otimes\tilde{e}_{22} + \tilde{e}_{22}e^{\tilde{\sigma}_{11}}\otimes\tilde{e}_{12}\tilde{e}_{02}\right)(\tilde{\omega}_{22}\otimes\tilde{\omega}_{22})\,\tilde{\Omega},\end{aligned}$$

$$\begin{aligned}&\Delta_{\xi_1\xi_2}(\tilde{e}_{1-2})\\ &= \tilde{e}_{1-2}\otimes e^{-\tilde{\sigma}_{22}} + e^{\tilde{\sigma}_{11}}\otimes\tilde{e}_{1-2} - \xi_2\tilde{e}_{2-2}e^{\tilde{\sigma}_{11}}\otimes\tilde{e}_{12}\,e^{-2\tilde{\sigma}_{22}}\\ &+\xi_2\left(\tilde{e}_{01}\otimes\tilde{e}_{02}e^{-\tilde{\sigma}_{22}} + \tilde{e}_{02}e^{\tilde{\sigma}_{11}}\otimes\tilde{e}_{01}\right.\\ &-\xi_2\tilde{e}_{12}\tilde{e}_{02}\tilde{\omega}_{22}e^{-\tilde{\sigma}_{22}}\otimes\tilde{e}_{02}e^{-\tilde{\sigma}_{22}}\\ &-\xi_2\tilde{e}_{02}e^{\tilde{\sigma}_{11}}\otimes\tilde{e}_{12}\tilde{e}_{02}\tilde{\omega}_{22}e^{-\tilde{\sigma}_{22}}\\ &\left.-\,\xi_2\tilde{e}_{02}e^{\tilde{\sigma}_{11}-\tilde{\sigma}_{22}}\otimes\tilde{e}_{12}\tilde{e}_{02}e^{-2\tilde{\sigma}_{22}}\right)\tilde{\Omega}\\ &+\xi_2^2\left(\tilde{e}_{01}\tilde{e}_{02}\otimes\tilde{e}_{22}e^{-\tilde{\sigma}_{22}} - \tilde{e}_{22}e^{\tilde{\sigma}_{11}}\otimes\tilde{e}_{01}\tilde{e}_{02}\right)\\ &\times(\tilde{\omega}_{22}\otimes\tilde{\omega}_{22})\,\tilde{\Omega} \quad (3.18)\\ &+\frac{\xi_2^2}{2}\left(\tilde{e}_{12}\otimes\tilde{e}_{22}e^{-\tilde{\sigma}_{22}} + \tilde{e}_{22}e^{\tilde{\sigma}_{11}}\otimes\tilde{e}_{12}\right)(\tilde{\omega}_{22}\otimes\tilde{\omega}_{22})\,\tilde{\Omega},\end{aligned}$$

$$\begin{aligned}&\Delta_{\xi_1}(\tilde{e}_{1-1})\\ &= \tilde{e}_{1-1}\otimes e^{-2\tilde{\sigma}_{11}} + 1\otimes\tilde{e}_{1-1}\\ &+\xi_1\left(\tilde{e}_{12}\otimes\tilde{e}_{1-2}e^{\tilde{\sigma}_{22}} - \tilde{e}_{1-2}\otimes\tilde{e}_{12}e^{-\tilde{\sigma}_{22}} + \tilde{e}_{01}\otimes\tilde{e}_{01}\right.\\ &+\xi_2\left(\tilde{e}_{01}\otimes\tilde{e}_{02}\tilde{e}_{12}\tilde{\omega}_{22}e^{-\tilde{\sigma}_{22}} - \tilde{e}_{02}\tilde{e}_{12}\tilde{\omega}_{22}\otimes\tilde{e}_{01}\right.\\ &+\tilde{e}_{01}\tilde{e}_{02}\tilde{\omega}_{22}\otimes\tilde{e}_{12}e^{-\tilde{\sigma}_{22}} + \tilde{e}_{12}\otimes\tilde{e}_{01}\tilde{e}_{02}\tilde{\omega}_{22}\\ &-\frac{1}{2}\tilde{e}_{12}\tilde{\omega}_{22}\otimes\tilde{e}_{12}e^{-\tilde{\sigma}_{22}} - \frac{1}{2}\tilde{e}_{12}\otimes\tilde{e}_{12}\tilde{\omega}_{22}\\ &\left.\left.-\tilde{e}_{2-2}\tilde{e}_{12}\otimes\tilde{e}_{12}e^{-\tilde{\sigma}_{22}} + \xi_2\tilde{e}_{02}\tilde{e}_{12}\tilde{\omega}_{22}\otimes\tilde{e}_{02}\tilde{e}_{12}\tilde{\omega}_{22}e^{-\tilde{\sigma}_{22}}\right)\right)\\ &\times\left(e^{-\tilde{\sigma}_{11}}\otimes e^{-2\tilde{\sigma}_{11}}\right). \quad (3.19)\end{aligned}$$

Here we use the notation $\xi_2\tilde{e}_{22} = e^{2\tilde{\sigma}_{22}} - 1$, $\tilde{\omega}_{22} := (e^{\tilde{\sigma}_{22}} + 1)^{-1}$, $\tilde{\Omega} := \Delta_{\xi_2}(\tilde{\omega}_{22}) = (e^{\tilde{\sigma}_{22}}\otimes e^{\tilde{\sigma}_{22}} + 1)^{-1}$. The twisted Hopf structure for the generators of subalgebra $\widetilde{\mathfrak{osp}}_2(1|2)$ can be found in [34].

4 Light-cone κ-deformation of the super-Poincaré algebra $\mathcal{P}(3,1|1)$

In order to propose the application of the deformed superalgebra $\mathcal{A}\in U_{\xi_1\xi_2}(\mathfrak{osp}(1|4))$ to the description of anti-de Sitter symmetries one should consider the real forms which leave the classical r-matrices (2.11) and (2.12) skew-Hermitian , i.e.

$$(r_1(\xi_1))^\star = -r_1(\xi_1), \quad (r_2(\xi_2))^\star = -r_2(\xi_2). \quad (4.1)$$

where the $\star$-conjugation is an antilinear (super-)antiautomorphism, and it defines the real form $\mathfrak{sp}(4;\mathbb{R})$ of $\mathfrak{sp}(4;\mathbb{C})\subset\mathfrak{osp}(1|4)$.

According to [34] we consider two versions of the $\star$-conjugation which satisfy the condition (4.1).

(i) The $\dagger$-conjugation is defined as follows:

$$e_{jk}^\dagger = -e_{jk}, \quad e_{0j}^\dagger = -\mathrm{i}e_{0j} \quad (j,k=\pm1,\pm2), \quad (4.2)$$

provided that

$$(ab)^\dagger = b^\dagger a^\dagger, \quad (a\otimes b)^\dagger = (-1)^{\deg a\,\deg b}\,a^\dagger\otimes b^\dagger \quad (4.3)$$

for any homogeneous elements a, $b\in U_{\xi_1\xi_2}(\mathfrak{osp}(1|4))$.

(ii) The $\ddagger$-conjugation we define by

$$e_{jk}^\ddagger = -e_{jk}, \quad e_{0j}^\ddagger = -e_{0j} \quad (i,j=\pm1,\pm2), \quad (4.4)$$

provided that

$$(ab)^\ddagger = (-1)^{\deg a\,\deg b}b^\ddagger a^\ddagger, \quad (a\otimes b)^\ddagger = a^\ddagger\otimes b^\ddagger \quad (4.5)$$

for any homogeneous elements a, $b\in U_{\xi_1\xi_2}(\mathfrak{osp}(1|4))$. From the condition that σ_{11} (see (3.6)) and $\tilde{\sigma}_{22}$ (see (3.14)) are Hermitian, it follows that the parameters ξ_1, ξ_2 are purely imaginary.

Let M_{AB} $(A,B=0,1,2,3,4)$ describe the rotation generators of the AdS superalgebra $\mathfrak{o}(3,2)\simeq\mathfrak{sp}(4;\mathbb{R})$ with the standard relations

$$[M_{AB},\,M_{CD}] = \mathrm{i}\,(g_{BC}\,M_{AD} - g_{BD}\,M_{AC} + g_{AD}\,M_{BC} - g_{AC}\,M_{BD}), \quad (4.6)$$

$$M_{AB} = -M_{BA}, \quad M_{AB}^\star = M_{AB} \quad (4.7)$$

where $g_{AB} = \mathrm{diag}(1, -1, -1, -1, 1)$. The Cartan–Weyl (CW) generators e_{jk} of $\mathfrak{sp}(4;\mathbb{R})$ can be realized in the terms of the generators M_{AB} as follows:

$$\begin{aligned} e_{1-2} &= \mathrm{i}\,(M_{42}+M_{21})\,, \quad e_{12} = \mathrm{i}\,(M_{02}+M_{32})\,, \\ e_{1-1} &= \mathrm{i}\,(M_{03}+M_{14})\,, \quad e_{2-1} = \mathrm{i}\,(M_{42}-M_{21})\,, \\ e_{-2-1} &= \mathrm{i}\,(M_{02}-M_{32})\,, \quad e_{2-2} = \mathrm{i}\,(M_{03}-M_{14})\,, \\ e_{11} &= \mathrm{i}\,(M_{34}+M_{04}+M_{13}-M_{01})\,, \\ e_{-1-1} &= \mathrm{i}\,(M_{34}-M_{04}-M_{13}-M_{01})\,, \\ e_{22} &= \mathrm{i}\,(M_{34}+M_{04}-M_{13}+M_{01})\,, \\ e_{-2-2} &= \mathrm{i}\,(M_{34}-M_{04}+M_{13}+M_{01})\,. \end{aligned} \tag{4.8}$$

Using these formulas one can write the boson (even) part of the classical r-matrix (2.10) in terms of the physical generators M_{AB}[4]:

$$\begin{aligned} & r^b_{1,2}(\xi_1,\xi_2) \\ &= \xi_1 \left(\frac{1}{2}\, e_{1-1}\wedge e_{11} + e_{1-2}\wedge e_{12}\right) + \frac{1}{2}\xi_2\, e_{2-2}\wedge e_{22} \\ &= \frac{1}{2}(\xi_2-\xi_1) \\ &\quad \times\Big(M_{14}\wedge(M_{34}+M_{04}) + M_{03}\wedge(M_{14}-M_{01})\Big) \\ &\quad -\frac{1}{2}(\xi_1+\xi_2) \\ &\quad \times\Big(M_{14}\wedge(M_{13}-M_{01}) + M_{03}\wedge(M_{34}+M_{04})\Big) \\ &\quad -\xi_1(M_{42}+M_{21})\wedge(M_{02}+M_{32}). \end{aligned} \tag{4.9}$$

There are two special cases, $\xi_1 = \xi_2$ and $\xi_1 = -\xi_2$. We are interested in the first case $\xi = \xi_1 = \xi_2$:

$$\begin{aligned} r^b(\xi) &:= r^b_{1,2}(\xi,\xi) \\ &= -\xi\,(M_{14}\wedge(M_{13}-M_{01}) + M_{03}\wedge(M_{34}+M_{04}) \\ &\quad + (M_{42}+M_{21})\wedge(M_{02}+M_{32}))\,. \end{aligned} \tag{4.10}$$

Introducing $\mu,\nu = 0,1,2,3$ and

$$M_{\mu 4} = R\,\mathcal{P}_\mu, \tag{4.11}$$

we obtain from (4.6) the basic relation of the $D = 4$ algebra AdS:

$$[\mathcal{P}_\mu,\mathcal{P}_\nu] = -\frac{1}{R^2}\, M_{\mu\nu}. \tag{4.12}$$

Using the physical AdS assignment of the generators $\{M_{AB}\} = (M_j, N_j, \mathcal{P}_j, \mathcal{P}_0)$, where $M_j = \frac{1}{2}\epsilon_{jkl}M_{kl}$, $N_j = M_{0j}$ $(j,k,l = 1,2,3)$ one can write the classical r-matrix (4.10) as follows:

$$\begin{aligned} r^b(\xi) &= \xi R\,(\mathcal{P}_1\wedge(N_1+M_2) + \mathcal{P}_2\wedge(N_2-M_1) \\ &\quad + \mathcal{P}_+\wedge N_3) + \xi\, M_3\wedge(N_2-M_1), \end{aligned} \tag{4.13}$$

[4] Compare with [13] where other relations between the physical AdS and CW bases were used.

where $\mathcal{P}_+ = \mathcal{P}_0 + \mathcal{P}_3$. Now we put $\xi = \frac{\mathrm{i}}{\kappa R}$ and perform the limit $R \to \infty$ (see (1.3)). In such a way we obtain the classical r-matrix (1.4) describing the light-cone κ-deformation of the Poincaré algebra ($\lim_{R\to\infty} \mathcal{P}_\mu = P_\mu$)

$$\begin{aligned} r^b_\kappa &:= \lim_{R\to\infty} r^b\left(\frac{\mathrm{i}}{\kappa R}\right) \\ &= \frac{\mathrm{i}}{\kappa}\Big(P_1\wedge(N_1+M_2) + P_2\wedge(N_2-M_1) + P_+\wedge N_3\Big), \end{aligned} \tag{4.14}$$

where the parameter κ is real, and the Poincaré algebra generators (M_j, N_j, P_i, P_0) satisfy the relations (1.2).

Similarly one can discuss the classical $\mathfrak{osp}(1|4)$ r-matrix (2.10) and its contraction limit. In order to obtain a finite result we put $\xi_1 = \xi_2$ and introduce in accordance with (4.2) and (4.4) the real $\mathfrak{osp}(1|4)$ super-charges as follows:

$$e_{0\pm k} = \sqrt{\mathrm{i}R}\, Q_{\pm k} \quad (k = \pm 1, \pm 2). \tag{4.15}$$

One gets the formula for the super-Jordanian classical $\mathfrak{osp}(1|4)$ r-matrix

$$\begin{aligned} r(\xi) &:= r_{12}(\xi,\xi) \\ &= r^b_{12}(\xi,\xi) - 2\mathrm{i}R\,\xi(Q_1\wedge Q_1 + Q_2\wedge Q_2), \end{aligned} \tag{4.16}$$

which leads in the limit (1.3) to the following super-Poincaré classical r-matrix:

$$\begin{aligned} r^{susy}_\kappa &:= \lim_{R\to\infty} r\left(\frac{\mathrm{i}}{\kappa R}\right) \\ &= r^b_\kappa + \frac{2}{\kappa}\,(Q_1\wedge Q_1 + Q_2\wedge Q_2)\,. \end{aligned} \tag{4.17}$$

The classical r-matrix (4.17) describes the superextension of the light-cone κ-deformation of the Poincaré algebra.

In order to describe the adjoint action of $\mathcal{P}(3,1)$ on the four real supercharges Q_α $(\alpha = \pm 1, \pm 2)$ it is convenient to introduce a 2-component complex Weyl basis. In terms of the Weyl basis $Q_1^{(\pm)} := Q_1 \pm \mathrm{i}Q_2$, $Q_2^{(\pm)} := Q_{-1} \pm \mathrm{i}Q_{-2}$ the commutation relations read as follows:

$$\begin{aligned} [M_j, Q^{(\pm)}_\alpha] &= -\frac{\mathrm{i}}{2}(\sigma_j)_{\alpha\beta}\, Q^{(\pm)}_\beta, \\ [N_j, Q^{(\pm)}_\alpha] &= \mp\frac{\mathrm{i}}{2}(\sigma_j)_{\alpha\beta}\, Q^{(\pm)}_\beta, \quad [P_\mu, Q^{(\pm)}_\alpha] = 0, \end{aligned} \tag{4.18}$$

and moreover

$$\begin{aligned} &\{Q^{(\pm)}_\alpha,\, Q^{(\pm)}_\beta\} = 0, \\ &\{Q^{(+)}_\alpha,\, Q^{(-)}_\beta\} = 2\,(\delta_{\alpha\beta}\, P_0 - (\sigma_j)_{\alpha\beta}\, P_j)\,, \end{aligned} \tag{4.19}$$

where σ_j $(j = 1,2,3)$ are 2×2 σ-matrices. The spinor $\mathbf{Q}^{(+)} := (Q_1^{(+)}, Q_2^{(+)})$ transforms as the left-regular representation and the spinor $\mathbf{Q}^{(-)} := (Q_1^{(-)}, Q_2^{(-)})$ provides the right-regular one.

Using the commutation relations (1.2) and (4.18), (4.19) it easy to check that the r-matrix (4.17) (and also (4.14)) is of the Jordanian type (1.5) and (1.6), where $h_{\gamma_0} \to iN_3$,

Drinfeld–Jimbo deformation $U_q(\mathfrak{osp}(1\|4))$	$\longrightarrow$ $\ln q = \frac{1}{\kappa R}$ $(R \to \infty)$	Standard κ-deformed $D = 4$ Poincaré superalgebra [15–17]
Jordanian type deformation $U_{\xi_1,\xi_2}(\mathfrak{osp}(1\|4))$	$\longrightarrow$ $\xi_1 = \xi_2 = \frac{\mathrm{i}}{\kappa R}$ $(R \to \infty)$	Light-cone κ-deformation of $D = 4$ Poincaré superalgebra

Fig. 1. Two different κ-deformations of the $D = 4$, $N = 1$ supersymmetries

$e_{\gamma_0} \to P_+$, etc. Therefore we can immediately read off the twisting two-tensor corresponding to this r-matrix [30] as an analog of the formulas (3.2)–(3.6). However we can obtain also a twisting two-tensor corresponding to the classical r-matrix (4.17) by applying the contraction AdS limit to the full Jordanian type twisting two-tensor of $\mathfrak{osp}(1|4)$. This full twist $F(\xi_1, \xi_2)$ (3.1) can be presented as follows:

$$\begin{aligned} F(\xi_1,\xi_2) &= \tilde{\mathfrak{F}}_{02}(\xi_2)\mathfrak{F}_{01}(\xi_1) \qquad (4.20)\\ &\quad\times\left(\tilde{F}_{\sigma_{22}} \exp\left(\xi_1 e_{1-2} \otimes e_{12} e^{-\sigma_{11}}\right) \tilde{F}^{-1}_{\sigma_{22}}\right) \tilde{F}_{\sigma_{22}} F_{\sigma_{11}}\\ &= \tilde{\mathfrak{F}}_{02}(\xi_2)\mathfrak{F}_{01}(\xi_1) \exp\left(\xi_1 e_{1-2} \otimes e_{12} e^{-\sigma_{11}-\tilde{\sigma}_{22}}\right) \tilde{F}_{\sigma_{22}} F_{\sigma_{11}}. \end{aligned}$$

Replacing here the mathematical generators e_{jk} by the physical ones M_{AB} and performing the contraction limit we obtain as a result the twisting two-tensor for the light-cone κ-deformation of the super-Poincaré algebra:

$$\begin{aligned} &F_\kappa(\mathcal{P}(3,1|1)) \qquad (4.21)\\ &:= \lim_{R\to\infty} F\left(\frac{\mathrm{i}}{\kappa R}, \frac{\mathrm{i}}{\kappa R}\right) = \mathfrak{F}_\kappa(Q_2)\mathfrak{F}_\kappa(Q_1) F_\kappa(\mathcal{P}(3,1)), \end{aligned}$$

where $F_\kappa(\mathcal{P}(3,1))$ is the twisting two-tensor of the light-cone κ-deformation of the Poincaré algebra $\mathcal{P}(3,1)$:

$$\begin{aligned} &F_\kappa(\mathcal{P}(3,1)) \qquad (4.22)\\ &:= e^{\frac{\mathrm{i}}{\kappa} P_1 \otimes (N_1+M_2) e^{-2\sigma_+}} \, e^{\frac{\mathrm{i}}{\kappa} P_2 \otimes (N_2-M_1) e^{-2\sigma_+}} \, e^{2\mathrm{i} N_3 \otimes \sigma_+} \end{aligned}$$

and the super-factors $\mathfrak{F}_\kappa(Q_\alpha)$ $(\alpha = 1, 2)$ are given by the formula

$$\begin{aligned} \mathfrak{F}_\kappa(Q_\alpha) &= \sqrt{\frac{(1+e^{\sigma_+}) \otimes (1+e^{\sigma_+})}{2(1+e^{\sigma_+} \otimes e^{\sigma_+})}}\\ &\quad\times\left(1 + \frac{2}{\kappa} \frac{Q_\alpha}{1+e^{\sigma_+}} \otimes \frac{Q_\alpha}{1+e^{\sigma_+}}\right), \qquad (4.23) \end{aligned}$$

$$\sigma_+ := \frac{1}{2} \ln\left(1 + \frac{1}{\kappa} P_+\right). \qquad (4.24)$$

Since $[N_1 + M_2, \sigma_+] = [N_2 - M_1, \sigma_+] = [N_1 + M_2, N_2 - M_1] = 0$, all three exponentials on the right side of (4.22) mutually commute and they can be written in any order. We add that the super-factors in (4.21) also mutually commute.

Using the twisting two-tensors (4.22) and (4.23) we can calculate the twisted coproducts and twisted antipodes for the generators of the Poincaré and super-Poincaré algebras. These formulas will be given in a future publication. It should be noted that the twisting functions (3.1), (4.22) and (4.23) satisfy the unitarity condition, i.e., for example,

$$F^\star_\kappa(\mathcal{P}(3,1)) = F^{-1}_\kappa(\mathcal{P}(3,1)). \qquad (4.25)$$

Therefore the twisted coproduct $\Delta_\kappa(x) := F_\kappa \Delta(x) F^{-1}_\kappa$ and the twisted antipode $S_\kappa(x)$ are real under the *-conjugation, i.e. $(\Delta_\kappa(x))^\star = \Delta_\kappa(x^\star)$, $S_\kappa((S_\kappa(x^\star))^\star) = x$.

5 Outlook

In this paper we studied the Jordanian type deformation of $\mathfrak{osp}(1|4)$ with the two deformation parameters ξ_1, ξ_2. If we interpret physically $\mathfrak{osp}(1|4)$ as the $D = 4$ AdS superalgebra, the parameters ξ_1 and ξ_2 are dimensionless and the role of the dimensionful parameter takes over the AdS radius R. The introduction of a $D = 4$ super-Poincaré limit requires the relation $\xi_1 = \xi_2 = \xi$ and the special contraction procedure described by the formula (1.3) with R-dependent single deformation parameter ξ. In such a way we obtain a new quantum deformation of the $D = 4$ super-Poincaré algebra with κ as the deformation parameter. Recalling [27] we can therefore introduce by the contraction procedure two different κ-deformations of the $D = 4$, $N = 1$ supersymmetries as represented in Fig. 1.

It is known that the light-cone κ-deformation of the $D = 4$ Poincaré algebra with the classical r-matrix satisfying CYBE (Classical Yang-Baxter Equation) can be extended to $D = 4$ conformal symmetries (see e.g. [12]). Analogously, the light-cone κ-deformation of $D = 4$ the Poincaré superalgebra can be obtained by studying the suitable Jordanian type deformation of $D = 4$ conformal superalgebra $\mathfrak{su}(2,2|1)$. The Jordanian type deformations of $\mathfrak{su}(2,2|1)$ and in particular the new embeddings of the κ-deformations of $D = 4$ super-Poincaré algebra are now under consideration.

Finally we would like to mention that the κ-deformation of the N-extended AdS supersymmetries can be described by the Jordanian type deformations of $\mathfrak{osp}(N|4)$. An outline of the mathematical framework describing the Jordanian type twist quantization of $\mathfrak{osp}(M|2n)$ has been given recently in [30].

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